MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2022/2023 (2220)**

## DSP5221 DISCRETE STRUCTURES & PROBABILITY

( DIT )

3 JULY 2023
2.30PM – 4.30PM
( 2 Hours )

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**INSTRUCTIONS TO STUDENT**

1. This Question paper consists of 4 pages, excluding the cover page and appendices.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.
4. Only **NON-PROGRAMMABLE** calculator is allowed.

**Please answer ALL questions and show the necessary working. Total marks is equal to 40.**

## Question 1

a) Show by truth table that the following statement formula is a tautology.

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$ (2.5 marks)

b) Consider the following function. (3 marks)

```
int curve(int y)
{
    if (y == 0)
        return 0;
    else
        return (curve(y – 1) + (y + 25));
}
```

Show the tracing process to derive the result by this function call: **curve (5)**.

c) Let the Universal Set, $U = \{x \mid x \text{ is a prime number less than } 25\}$,

set $M = \{2,3,11,13,17\}$, set $N = \{2,7,11,13,17\}$ and set $O = \{11,17,19\}$.

i) Construct a Venn Diagram to represent the sets above. (3 marks)

ii) What are the elements of $M - (N \cup O)$? (0.5 mark)

iii) What are the elements of $\overline{(M \cup N \cup O)}$? (0.5 mark)

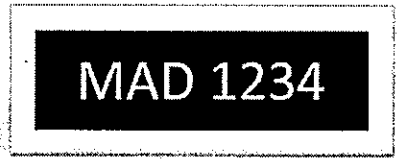
iv) Find $|M \cup N \cup O|$. (0.5 mark)

**[10 marks]**

**Continued ...**

## Question 2

a) Maria would like to buy a car registration plate number from Road Transport Department Malaysia (JPJ Malaysia) which consists of 3 alphabets and followed by 4 digits. The first alphabet and first digit must be "M" and "1" respectively. The rest of the alphabets and digits can be repeated but should not the same as the first alphabet and first digit. Count how many selections are available for Maria. (1.5 marks)




b) Table 2 below shows the number of staff who works in the accounting and sales department for three different companies. If a staff has been selected randomly, determine the probability of he/she is from Company 3 given that he/she is working in sales department. (2.5 marks)

**Table 2**

| Company | Department | |
|---|---|---|
| | Accounting | Sales |
| Company 1 | 4 | 16 |
| Company 2 | 3 | 31 |
| Company 3 | 12 | 43 |

c) On average, Maple will use the Foodpanda app for 1.1 time to order food in a day. Determine the probability of she will use Foodpanda app for at least 6 times to order food in a week. (2.5 marks)

d) The pitaya grows in Multi-Rich Pitaya Dragon Fruit Farm is approximately between 302 and 650 grams. On average, the weight is normally distributed with a mean of 320 grams and a standard deviation of 120 grams. Calculate the probability of a pitaya which is between 302 and 650 grams. (3.5 marks)

**[10 marks]**

**Continued ...**

## Question 3

a) Let X denote the number of times a photocopy machine will malfunction: 0, 1, 2, or 3 times, on any given month. Let Y denote the number of times a technician is called on an emergency call. The joint probability mass function (pmf), $p(x, y)$ is presented in the Table 3 below:

**Table 3**

| $y$ | $x$ | | | | $p_Y(y)$ |
|---|---|---|---|---|---|
| | **0** | **1** | **2** | **3** | |
| **0** | 0.15 | 0.30 | 0.05 | 0 | 0.50 |
| **1** | 0.05 | 0.15 | 0.05 | 0.05 | 0.30 |
| **2** | 0 | 0.05 | 0.10 | 0.05 | 0.20 |
| $p_X(x)$ | 0.20 | 0.50 | 0.20 | 0.10 | 1.00 |

i) Find the probability $P(Y > X)$. (1.5 marks)

ii) Find $Cov(X, Y)$. (4.5 marks)

b) The continuous random variable x has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2), & 0 < x \leq 3 \\ 3k, & 3 < x \leq 4 \\ 0, & otherwise \end{cases}$$

where $k$ is a constant. Show that $k = \frac{1}{9}$. (4 marks)

**[10 marks]**

**Continued ...**

## Question 4

a) Check if the graphs in Figure 1 and Figure 2 have an Euler circuit. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one. (3 marks)

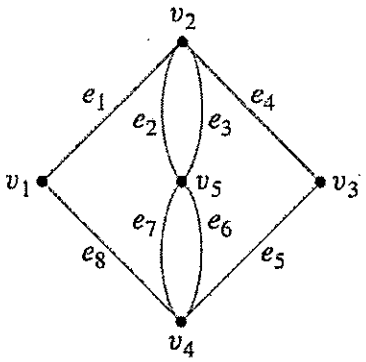



Figure 1

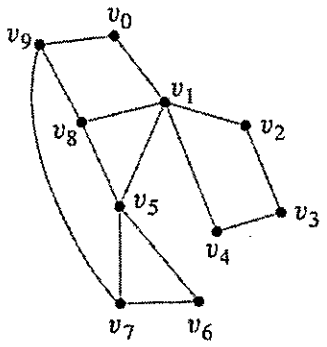



Figure 2

b) Build a binary search tree for the numbers: 25, 17, 9, 20, 33, 13, and 30. (3 marks)

c) Draw the state diagrams for the finite-state machines with these state table as shown in Table 4.

**Table 4**

| State | Input | |
|---|---|---|
| | 0 | 1 |
| $s_0$ | $s_1, 0$ | $s_0, 0$ |
| $s_1$ | $s_2, 1$ | $s_0, 1$ |
| $s_2$ | $s_0, 0$ | $s_3, 1$ |
| $s_3$ | $s_1, 1$ | $s_2, 0$ |

(4 marks)

**[10 marks]**

**End of Page.**

# APPENDICES

## Rules of Inference

| Rules of Inference | Name |
|---|---|
| $\frac{p}{\therefore p \vee q}$ | Addition |
| $\frac{p \wedge q}{\therefore p}$ | Simplification |
| $\frac{p \quad p \rightarrow q}{\therefore q}$ | Modus Ponens |
| $\frac{\sim q \quad p \rightarrow q}{\therefore \sim p}$ | Modus Tollens |
| $\frac{p \quad q}{\therefore p \wedge q}$ | Conjunction |
| $\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$ | Hypothetical Syllogism |
| $\frac{p \vee q \quad \sim p}{\therefore q}$ | Disjunctive Syllogism |
| $\frac{p \vee q \quad \sim p \vee r}{\therefore q \vee r}$ | Resolution |

## Binomial Distribution

| | | |
|---|---|---|
| Binomial probability formula: | | $P(X = x) = \binom{n}{x} p^x q^{n-x}$ |
| Key formula for Binomial Probability Distribution<br>If $p \leq 0.5$, | i. | $P(X = r) = B(r) - B(r-1)$ |
| | ii. | $P(X \geq r) = 1 - B(r-1)$ |
| | iii. | $P(X \leq r) = B(r)$ |
| | iv. | $P(X > r) = 1 - B(r)$ |
| | v. | $P(X < r) = B(r-1)$ |
| | vi. | $P(a \leq X \leq b) = B(b) - B(a-1)$ |
| If $p > 0.5$ | i. | $P(X = r) = B(n-r) - B(n-r-1)$ |
| | ii. | $P(X \geq r) = B(n-r)$ |
| | iii. | $P(X \leq r) = 1 - B(n-r-1)$ |
| | iv. | $P(a \leq X \leq b) = B(n-a) - B(n-b-1)$ |

## Poisson Distribution

| Poisson probability formula: | $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ |
|---|---|

Key formula for Poisson Probability Distribution:

If $\mu = \lambda$,

i. $P(X = r) = Poi(r) - Poi(r-1)$
ii. $P(X \geq r) = 1 - Poi(r-1)$
iii. $P(X \leq r) = Poi(r)$
iv. $P(X > r) = 1 - Poi(r)$
v. $P(X < r) = Poi(r-1)$
vi. $P(a \leq X \leq b) = Poi(b) - Poi(a-1)$

## Normal Probability Distribution

$z$-value (observed value) for an $x$ value : $z = \frac{x-\mu}{\sigma}$

## Discrete Random Variable

| Mean: $\mu = \sum xP(X = x)$ | Standard deviation: $\sigma = \sqrt{\sum x^2 P(X = x) - \mu^2}$ |
|---|---|
| Covariance: $\sigma_{XY} = E(XY) - E(X)E(Y)$ | Correlation: $\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ |

## Continuous Random Variable

| Expected Value: $E(X) = \mu_X = \int_{-\infty}^{\infty}\int x \cdot f(x,y)dxdy$ | Variance: $Var(X) = \sigma_X^2 = E(X^2) - [E(X)]^2$ |
|---|---|
| Covariance: $\sigma_{XY} = E(XY) - E(X)E(Y)$ | Correlation: $\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ |